BULK VISCOUS STRING COSMOLOGY WITH A TIME-VARYING COSMOLOGICAL CONSTANT: A MATHEMATICAL INVESTIGATION

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Abstract

Einstein field equations with cosmological constant is considered in the presence of bulk viscosity in Bianchi type-I universe. Solutions of the field equations are obtained by assuming the conditions: the bulk viscosity is proportional to the expansion scalar, $\xi \propto \theta$, expansion scalar is proportional to shear scalar $\theta \propto \sigma$, and Λ is proportional to Hubble parameter $\Lambda \propto H$ .The corresponding interpretations of the cosmological solutions are also discussed.

1. Introduction

Study of cosmology is the mathematical representation of the entire cosmos i.e. the study of starting, growth, and its fate. Additionally, cosmological models address the vast structure of the cosmos. It also explains how matter and energy are distributed throughout the cosmos and their characteristics. A collection of galaxies makes up the cosmos. In the 1930s, astronomers became aware that some galaxy clusters were spinning more quickly. It means there must be something else, something which we can't see, which adds gravity and acts like glue. That mysterious substance which acts like glue is "dark matter". In other words, our solar system, galaxies, and clusters of galaxies gripped together by something. This unseen glue that holds stars, dust, and gas together in a galaxy is known as dark matter, or enigmatic stuff. Scientists discovered that our cosmos is growing while they were researching it. However, the cosmos shouldn't be expanding so quickly if it is composed solely of the planets, stars, galaxies, and other known objects. Therefore, something must be causing the cosmos to expand more quickly. The nature of this energy is unknown to us.

Furthermore, we have no idea where it originates. However, it is clearly present.

Scientists choose the term "dark energy" for this substance.

Our Universe is made up of three main components:

68% Dark energy, 27% Dark matter, 5% Ordinary matter

Our universe is very vast. The mystery of cosmosfascinated humans from long time ago.

Is our universe limitless or finite?

Is the cosmosisequivalent and uniform?

Weather the universe is expanding?

What is the starting and what will be the end of this mysterious cosmos?

These are a few important questions which cosmologists are trying to answer from millions of years. In the history of cosmology, Newton's law of gravitation is the turning point. Cosmos means universe and logos means science or study therefore; cosmology is the study of universe. Newton's law of gravitation is known as milestone in the history of cosmology. Einstein created the general theory of relativity (GR) in 1915, and it has since been used to explain a number of cosmic phenomena.

The problem of cosmological constant is one of the most salient and unsettled problem in cosmology and the origin of our universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still unknown. Solutions to the field equations may also be generated by the law of variation of the scale factor which was proposed by Pavon [3]. In recent years cosmologists have been interested in constructing string cosmological models of the universe. The concept of string theory is developed to describe events at the early stages of the universe. It is believed that strings may be one source of density perturbation, and that are required for the formation of large-scale structure of the universe. Therefore, it is a subject of considerable interest of cosmologists to study cosmic strings in the framework of general relativity. The general relativistic formalism of cosmic strings is given by Letelier [8] and Stachel [4]. The gravitational effect of cosmic strings has been extensively discussed by Gott [5] and Letelier [9] in general relativity. The string cosmological models with magnetic field are investigated by Chakraborty [15].Kibble [16] studied the topology of cosmic domains and strings, Wang [17] discussed Bianchi Type-III string cosmological model with bulk viscosity and magnetic field.Bali and Upadhaya [14] investigated LRS Bianchi type-I bulk viscous fluid string cosmological model in general relativity.Raj Bali & A. Pradhan [13] have studied Bianchi type – III string cosmological models with time dependent Bulk Viscosity and Adhav et. al [6] have studied non-existence of Bianchi type-V string cosmological model with bulk viscous fluid in general relativity.Also H. Zhang et. al [29] studied the Friedmann cosmology on codimension 2 brane with time dependent tension. In this model the effective cosmological constant is free of the absolute value of the brane tension.Many authors examined different cosmological models using different conditions [31-37]

In this paper, we study Bianchi type-I space time with variable cosmological constant Λ. To obtained an explicit solution, we assume that coefficient of bulk viscosity is proportional to expansion scalar $\xi \propto \theta$, expansion scalar is proportional to shear scalar $\theta \propto \sigma$ and cosmological term proportional to the Hubble parameter $\Lambda \propto H$.

2. Metric and field equations

Consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \quad (1)$$

where A, B and C are functions of time t.

The energy momentum tensor for a cloud of strings with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}) \quad (2)$$

$$\text{Where} \quad \rho=\rho_p+\lambda \quad (3)$$

is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the rest energy density of the particles, $\theta = u^i_{;i}$ is the scalar of expansion, ξ is the coefficient of bulk viscosity and λ is the tension density of the cloud of strings. The u^i is the cloud four-velocity vector and ξ^i represents the direction of anisotropy, i.e. the direction of strings. They satisfies the relation

$$u^i u_i = -x^i x_i = -1, u^i x_i = 0 \quad (4)$$

The expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (5)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\right) \quad (6)$$

where H is the Hubble parameter.

Einstein's field equations with 8πG=1 and variable cosmological term Λ(t) in suitable units are

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - \Lambda(t)g_{ij}$$

The Ricci scalar is defined by

$$\therefore R = -2\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right)$$

For the metric (1), Einstein's field equations can be written as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\xi\theta-\Lambda \quad (7)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\xi\theta-\Lambda \quad (8)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\xi\theta-\Lambda \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=\lambda-\rho \quad (10)$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad (11)$$

Where dots on A, B and C denote the ordinary differentiation with respect to t.

From equation (11), we have

$A = k_1 B$ (12)

In order to obtain more general solution, we assume that coefficient of bulk viscosity is proportional to the expansion scalar [10], [17], [21]

i.e. $\xi = k\theta$ (13)

Now there are four independent equations (7), (10), (12) and (13) in six unknowns B, C, ξ, Λ, ρ and λ. Thus two more relations are needed to solve the system completely. We assume that the scalar of expansion is proportional to shear scalar $\theta \propto \sigma$. It is believed that evolution of one parameter should also be responsible for the evolution of the others (Vishwakarma 2005) [10], [23], [25]. In general, the above condition give rise to

$$B = C^n \quad (14)$$

Where n is a constant and the second condition is

$$\Lambda = aH \quad (15)$$

where H is the Hubble parameter and Λ is not a pure constant but rather decreases continuously with cosmic time. Also, Λ vary in such a way that energy is conserved (Arbab I Arbab 1997).

3. Solution of field equations

Substituting equation (15) into equation (5) and using equation (12), we get

$$\theta = (2n+1)\frac{\dot{C}}{C} \qquad (16)$$

$$\xi = k(2n+1)\frac{\dot{C}}{C} \qquad (17)$$

Using equation (5), (15), (16) & (17) , eq. (7) becomes

$$\left(\frac{\dot{C}}{C}\right)^{\cdot} + \left\{1 + \frac{n^2 - k(2n+1)^2}{n+1}\right\}\frac{\dot{C}^2}{C^2} = \frac{-a(2n+1)}{3(n+1)}\frac{\dot{C}}{C}$$

Simplifying, we get

$$\log\left(\frac{\dot{C}}{C}\right) + \left[\frac{n^2+n+1-k\left(2n+1\right)^2}{n+1}\right]\log C = -\frac{a(2n+1)t}{3(n+1)} + k_2$$

$$\log\left\{\left(\frac{\dot{C}}{C}\right)C^{\left(\frac{n^2+n+1-k\left(2n+1\right)^2}{n+1}\right)}\right\} = -\frac{a(2n+1)t}{3(n+1)} + k_2$$

$$\dot{C}C^{\left(\frac{n^2-k\left(2n+1\right)^2}{n+1}\right)} = \exp\left\{-\frac{a(2n+1)t}{3(n+1)} + k_2\right\}$$

Therefore $C = \left[\frac{n^2+n+1-k\left(2n+1\right)^2}{n+1}\left\{-\frac{3(n+1)}{a(2n+1)}\exp\left(-\frac{a(2n+1)t}{3(n+1)} + k_2\right) + k_3\right\}\right]^{\left(\frac{n+1}{n^2+n+1-k\left(2n+1\right)^2}\right)}$

And

$$\dot{C} = \left[\frac{n^2+n+1-k\left(2n+1\right)^2}{n+1}\left\{-\frac{3(n+1)}{a(2n+1)}\exp\left(-\frac{a(2n+1)t}{3(n+1)} + k_2\right) + k_3\right\}\right]^{\left(\frac{-n^2+k\left(2n+1\right)^2}{n^2+n+1-k\left(2n+1\right)^2}\right)}$$
$$\exp\left(-\frac{a(2n+1)t}{3(n+1)} + k_2\right)$$

$$\frac{\dot{C}}{C} = \left[\frac{\left(n^2+n+1\right) - k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)} + k_2\right) + k_3\right\}\right]^{-1}\exp\left(\frac{-a(2n+1)t}{3(n+1)} + k_2\right)$$

Therefore, the line element (1), becomes

$$ds^2 = -dt^2 + \left(k_1^2 dx^2 + dy^2\right)$$

$$\cdot\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{\frac{2n(n+1)}{(n^2+n+1)-k(2n+1)^2}}$$

$$+\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{\frac{2(n+1)}{(n^2+n+1)-k(2n+1)^2}} dz^2$$

The energy density ρ, the string tension density λ, coefficient of bulk viscosity ξ, the scalar of expansion θ, the shear scalar σ, Hubble parameter H, the particle density ρ_p, cosmological constant Λ are respectively given by

$$\theta = (2n+1)\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-1}$$

$$.\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

$$\xi = k(2n+1)\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-1}$$

$$.\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

$$\rho = 3k(2n+1)^2\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-2}$$

$$.\exp 2\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

$$H = \frac{(2n+1)}{3}\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-1}$$

$$.\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

$$\Lambda = \frac{a(2n+1)}{3}\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-1}$$

$$.\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

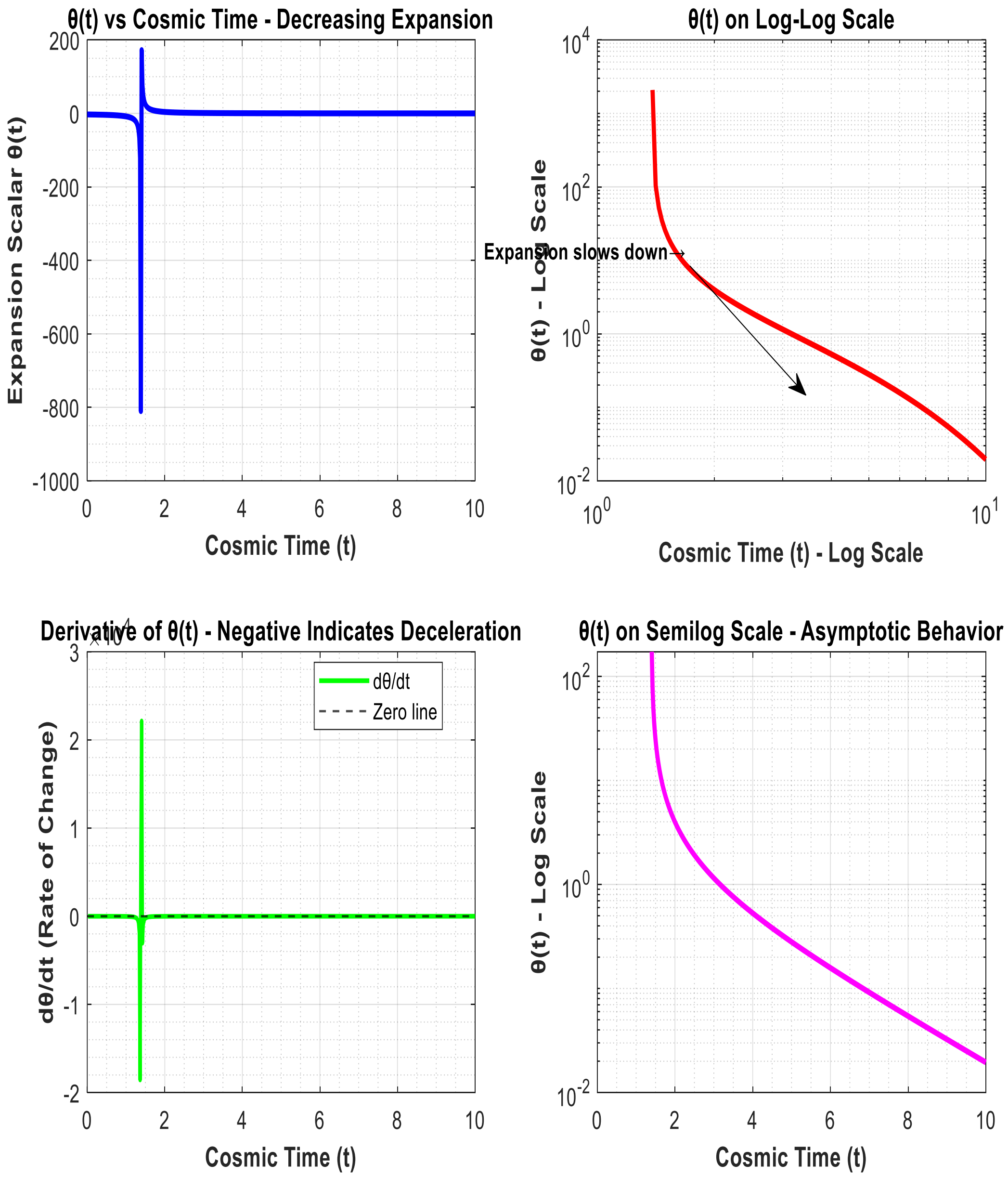
$$\rho_p = -n(n+2)\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-2}$$
$$.\exp 2\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

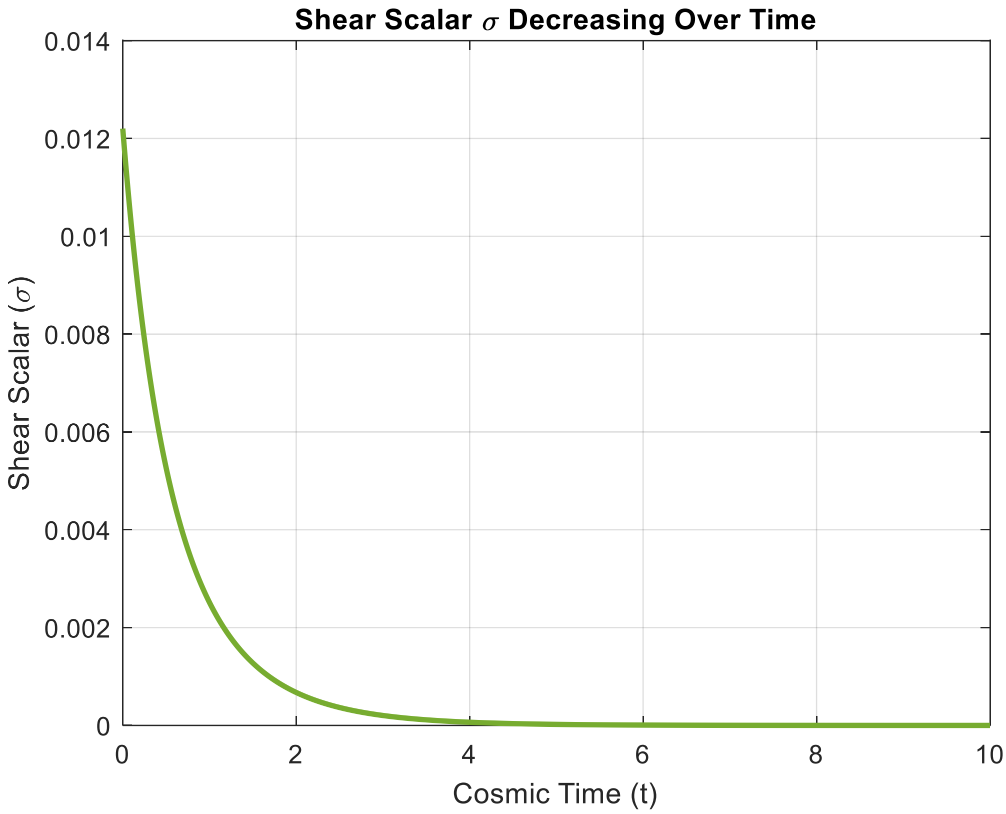
$$\lambda = \left\{n^2+2+3k(2n+1)^2\right\}\left[\frac{n^2+n+1-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-2}$$
$$.\exp 2\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

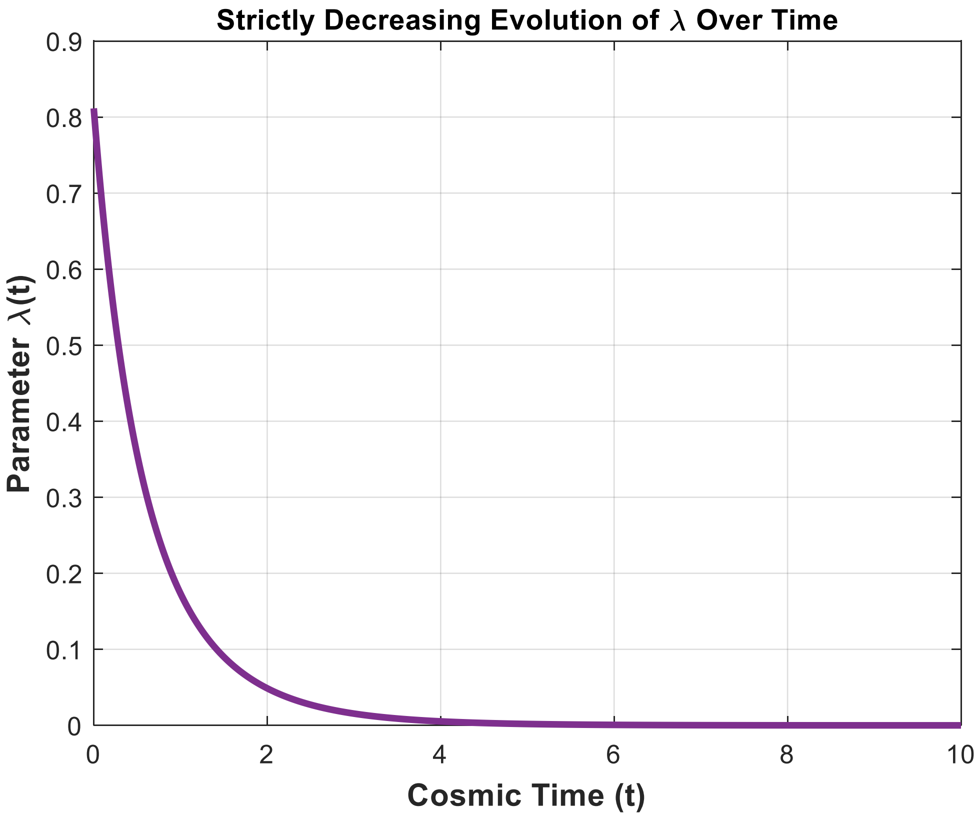
$$\sigma = \sqrt{\frac{n^2-2n+1}{3}}\left[\frac{\left(n^2+n+1\right)-k(2n+1)^2}{(n+1)}\left\{\frac{3(n+1)}{-a(2n+1)}\exp\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{-2}$$
$$.\exp 2\left(\frac{-a(2n+1)t}{3(n+1)}+k_2\right)$$

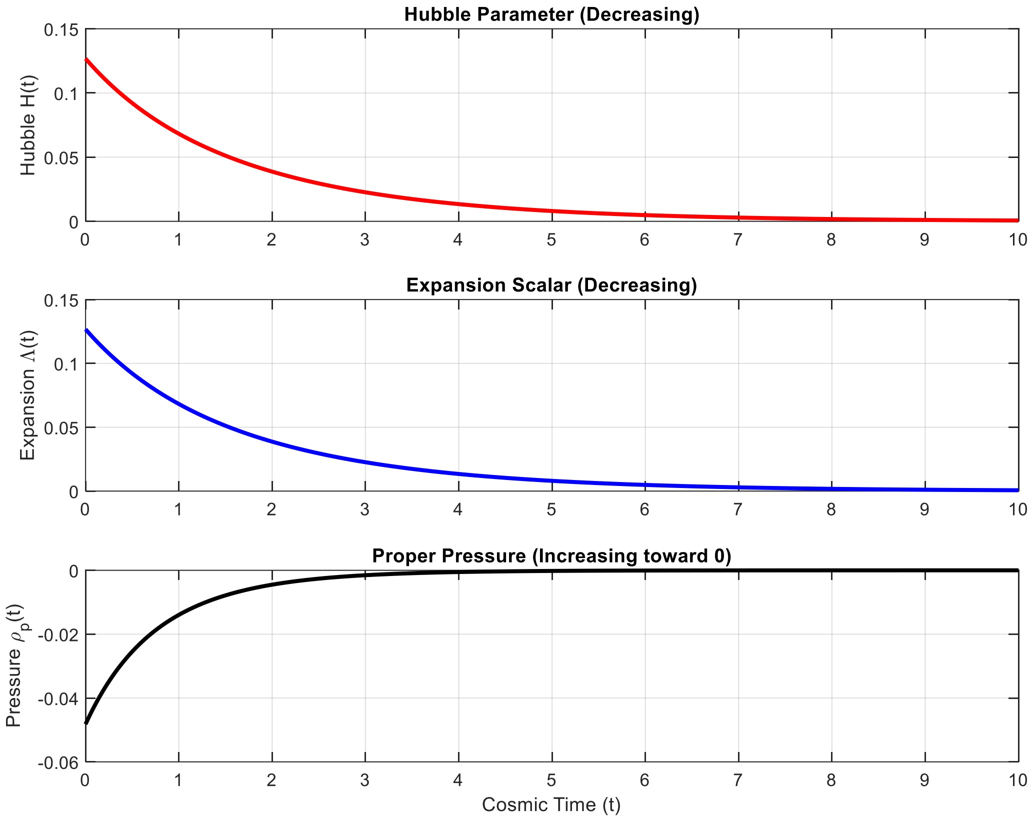
The volume V of the model is given by

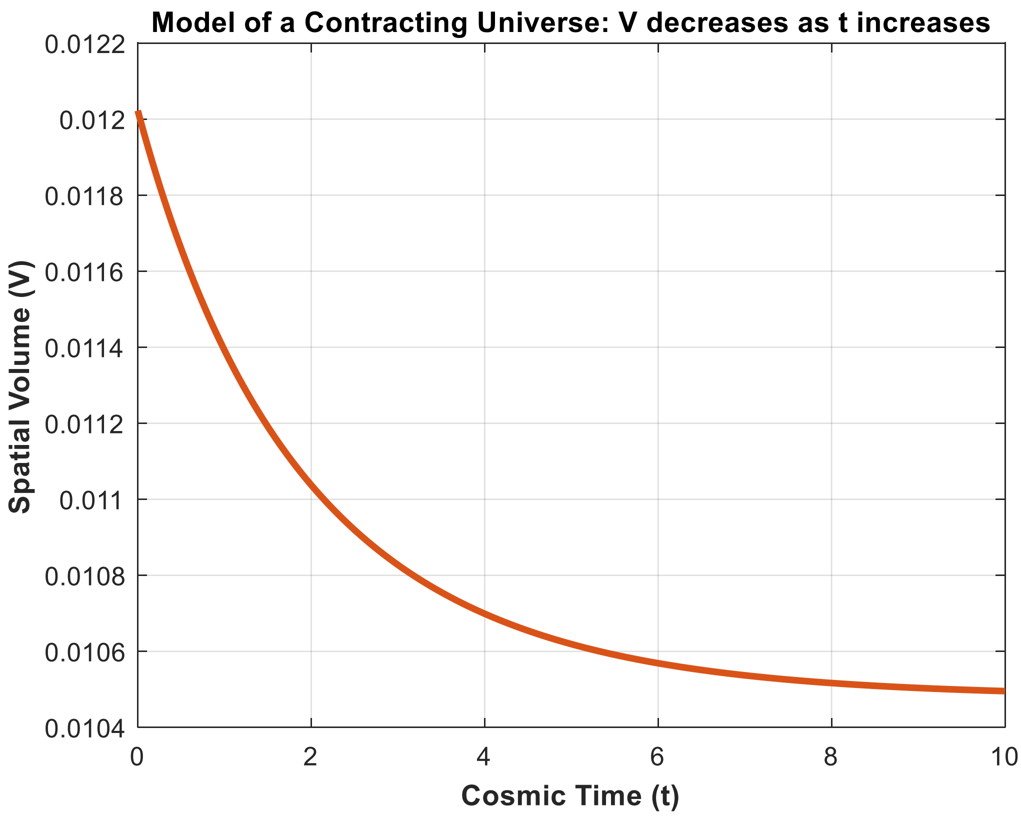
$$V = k_1\left[\frac{n^2+n+1-k\left(2n+1\right)^2}{n+1}\left\{-\frac{3(n+1)}{a(2n+1)}\exp\left(-\frac{a(2n+1)t}{3(n+1)}+k_2\right)+k_3\right\}\right]^{\left(\frac{(n+1)(2n+1)}{n^2+n+1-k\left(2n+1\right)^2}\right)}$$

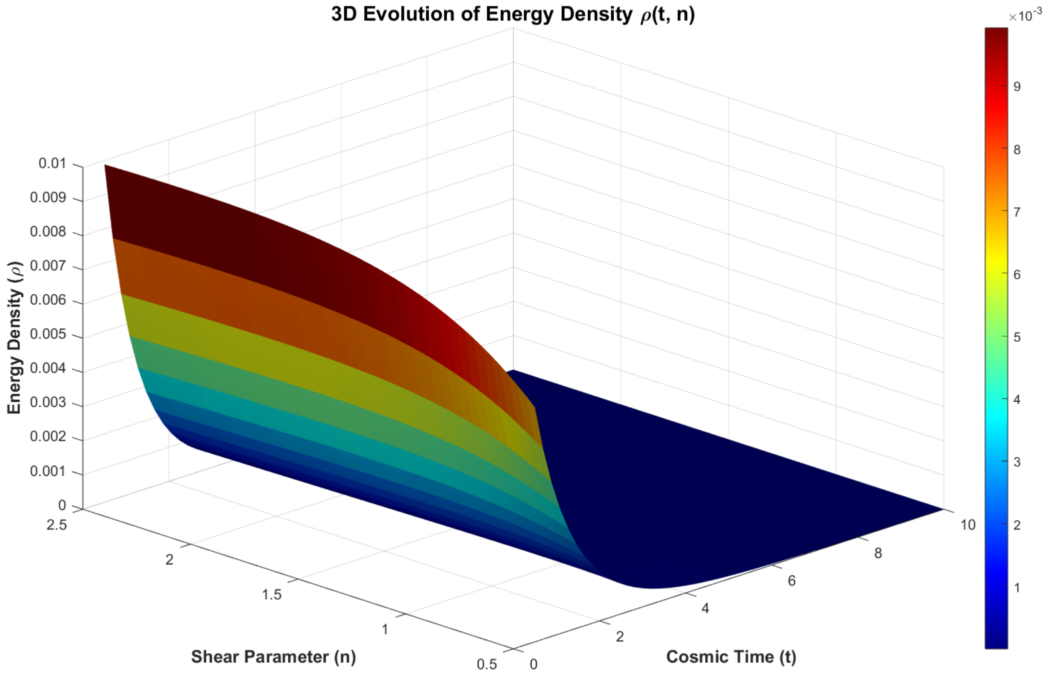

θ(t) vs Cosmic Time - Decreasing Expansion
Expansion Scalar θ(t)
Cosmic Time (t)
θ(t) on Log-Log Scale
θ(t) - Log Scale
Expansion slows down→
Cosmic Time (t) - Log Scale
Derivative of θ(t) - Negative Indicates Deceleration
dθ/dt (Rate of Change)
dθ/dt
Zero line
Cosmic Time (t)
θ(t) on Semilog Scale - Asymptotic Behavior
θ(t) - Log Scale
Cosmic Time (t)


Shear Scalar σ Decreasing Over Time
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
Shear Scalar (σ)
0
2
4
6
8
10
Cosmic Time (t)


Strictly Decreasing Evolution of λ Over Time
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Parameter λ(t)
0
2
4
6
8
10
Cosmic Time (t)


Hubble Parameter (Decreasing)
Hubble H(t)
Expansion Scalar (Decreasing)
Expansion Λ(t)
Proper Pressure (Increasing toward 0)
Pressure ρp(t)
Cosmic Time (t)


Model of a Contracting Universe: V decreases as t increases
Spatial Volume (V)
Cosmic Time (t)



4.Results and Discussion

In summary,

(i) It is clear from the final result that as time t decreases, the scalar of expansion θ increases and when the time t increases, the scalar of expansion θ decreases. Thus, rate of expansion slows down with the increase in time.

(ii) Energy density ρ decreases as the time t increases and ρ increases as the time t decreases. Therefore, the model describes a shearing non-rotating continuously expanding universe with a big-bang start.

(iii) Also λ, ρ_p, ξ, Λ and σ^2 decreases as time t increases. As $t \to \infty$ the volume becomes constant whereas λ, ρ_p, ξ, Λ and σ^2 tend to zero. Therefore, the model would essentially give an empty universe for a large value of t.

(iv) Since $\lim_{t\to\infty} \frac{\sigma}{\theta} \neq 0$, therefore model does not approach isotropy for large value of t.

(v) As the time t increases, the spacial volume V decreases. The rate of expansion slows down with the increase in time.

In summary, we have studied Bianchi type-I string cosmological model with bulk viscosity. Solution of field equations are obtained by assuming the condition: the bulk viscosity is proportional to the expansion scalar, $\xi \propto \theta$, expansion scalar is proportional to shear scalar $\theta \propto \sigma$ and Λ is proportional to Hubble parameter [10, 17, 28]. Then the cosmological model for a string cosmology with bulk viscosity is obtained. The physical and geometric aspects of the model are also discussed. The model describes a shearing non-rotating continuously expanding universe with a big-bang start.

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